

The CHARACTERISTICS of a FUNCTION

Definitions:

variable:

A letter or symbol, such as a , b , x , or n , that represents a number.

Eg. In the formula $A = l \times w \rightarrow A, l$ and w represent the Area, length and width of a rectangle.

independent variable:

In a relation, a variable whose values may be chosen and upon which the values of the other variable depend. Often represented by x .

dependent variable:

In a relation, the variable whose value depends on the value of another variable. Often represented by y .

FUNCTION:

A relation in which there is only one value of the dependent variable for each value of the independent variable. That is, for each x -value, there is only one y -value.

domain:

The set of all values for which the independent variable is defined.

(x)

range:

The set of all values of the dependent variable. All such values are determined from the values in the domain.

(y)

mapping diagram:

A drawing with arrows to show the relationship between each value of x and the corresponding values of y .

set notation:

A way of writing a group of items or numbers within curly brackets, $\{ \}$.

()

Example #1: Connecting Ordered Pairs and Tables of Values with Functions

For each relation, determine:

- (i) the domain and the range.
- (ii) whether or not it is a function; justify your answer.

a)

Number of Days in the Month, x	0	2	2	4	4
Minimum Daily Temperature ($^{\circ}\text{C}$), y	0	2	-2	-4	4

Domain = $\{0, 2, 4\}$

Range = $\{-4, -2, 0, 2, 4\}$

The relation is ***NOT*** a function because not each x -value has only one y -value.

The x -value **2** has two different y -values;
the x -value **4** has two different y -values.

b) $G(x, y) = \{(-1, -3), (0, 1), (1, 5), (2, 9)\}$

Domain = $\{-1, 0, 1, 2\}$

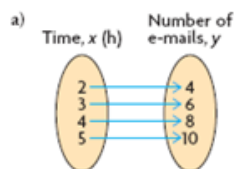
Range = $\{-3, 1, 5, 9\}$

The relation **is** a function because each x -value has only one unique y -value.

Example #2: Connecting Mapping Diagrams with Functions

For each relation, determine:

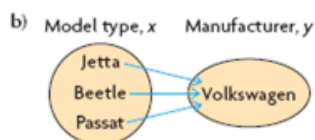
- (i) the domain and the range.
- (ii) whether or not it is a function; justify your answer.



Domain = {2, 3, 4, 5}

Range = {4, 6, 8, 10}

The relation **is** a function.
Each x -value has only one y -value.

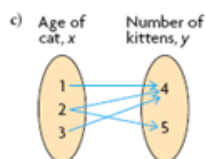


$D = \{\text{Jetta, Beetle, Passat}\}$

$R = \{\text{Volkswagen}\}$

The relation **is** a function. Each x -value has only one y -value.

If I know the model of car, I can predict the manufacturer.



$D = \{1, 2, 3\}$

$R = \{4, 5\}$

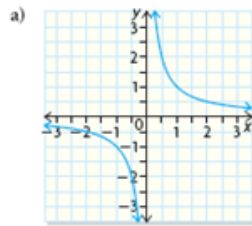
The relation is **NOT** a function. An x -value has more than one y -value.

Can't predict the # of kittens born from knowing the age of the mother.

Example #3: Connecting Graphs with Functions

For each relation, determine:

- the domain and the range, using *real numbers*.
- whether or not it is a function; justify your answer.



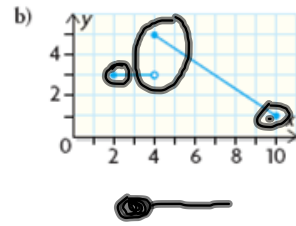
$$D = \{x \in \mathbb{R} \mid x \neq 0\}$$

→ x can be any real number except 0

$$R = \{y \in \mathbb{R} \mid y \neq 0\}$$

→ y can be any real number except 0

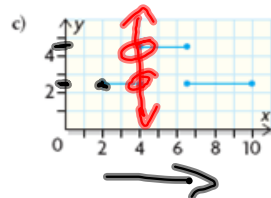
The relation **is** a function. Each x has only one y -value.



$$D = \{x \in \mathbb{R} \mid 2 \leq x \leq 10\}$$

$$R = \{y \in \mathbb{R} \mid 1 \leq y \leq 5\}$$

The relation **is** a function. Each x has only one y -value. ✓



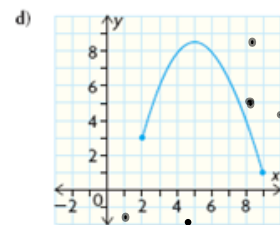
$$D = \{x \in \mathbb{R} \mid 2 \leq x \leq 10\}$$

$$R = \{y \in \mathbb{R} \mid y = 2.5, 4.5\}$$

The relation is **NOT** a function. An x -value has more than one y -value.



VERTICAL LINE TEST: A relation is **NOT** a function if at least one vertical line drawn through the graph of the relation passes through two or more points.



$$D = \{x \in \mathbb{R} \mid 2 \leq x \leq 9\}$$

$$R = \{y \in \mathbb{R} \mid 1 \leq y \leq 8.5\}$$

The relation **is** a function. Each x has only one y -value.

